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REDUCING UNCERTAINTY IN ENERGY ANALYSIS

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ABSTRACT

Each of the two principal methodologies for energy analysis has drawbacks which may severely limit the accuracy of results. A method for combining the two methods is presented, and is shown to yield results more accurate than either method used independently.

INTRODUCTION

This paper discusses two methods for calculating the energy cost of individual goods and services, or more complex systems. It presents a method for combining them in a way that minimizes uncertainty in the result.

The "energy cost" of an item is defined as the total energy required, directly and indirectly, to produce it. For example, the energy cost of a beer can includes not only the energy consumed by the can manufacturer, but also that required to smelt the aluminum, to mine the bauxite, plus all the energy needed to transport these intermediate products.

Energy cost is usually calculated using one of two methods. The first, called process analysis, starts with the final product and identifies direct process inputs, then the inputs to those products, etc. This "tree" of process inputs is shown in Fig. 1. The energy cost is obtained as the sum of the direct energy inputs at each juncture.*

The other method, based on input-output analysis starts with a description of an N-sector economic system producing N types of goods and services. The energy cost of all are determined simultaneously as the solution of a set of N linear equations.**

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*The direct energy input to a process is the enthalpy of the fuel consumed in that process. For examples of process analysis see Hannon (1973).

**The adaptation of economic input-output theory to energy analyses is described by Bullard and Herendeen (1975).

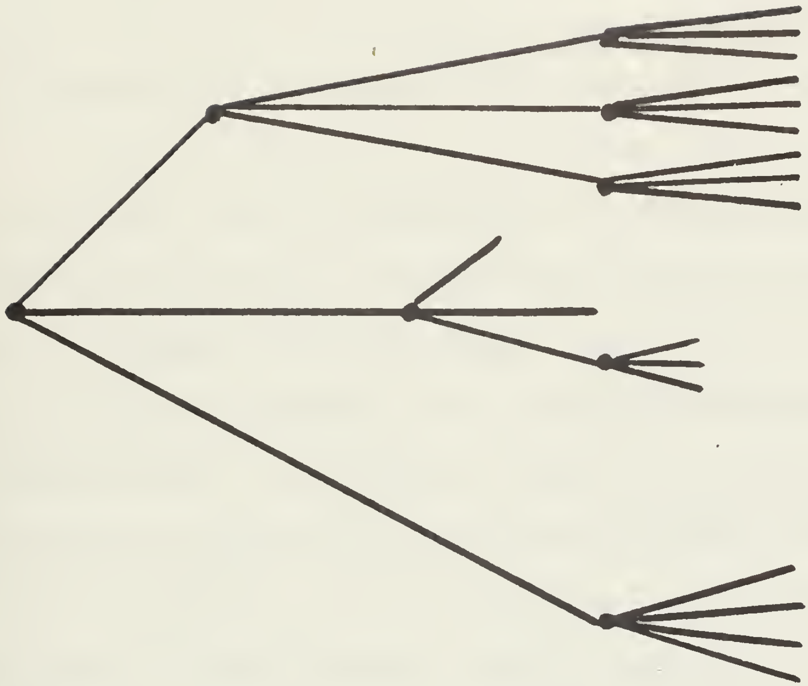

Figure 1. The PROCESS ANALYSIS "TREE"

The data required for each method are identical; the technology* for producing all types of goods and services must be specified. The energy cost per unit of output in each sector or energy-intensity is a function of this specification alone.

Process analyses are tedious and are usually truncated after only a few steps. This is usually done to limit data requirements to those describing production of only the most important inputs to a process. Truncation errors are unknown. Input-output (I-O) analysis avoids the truncation problem, as it includes all levels of inputs and feedback loops. It is limited, however, by the degree of disaggregation of the model which is necessarily general and not problem-specific. For example, I-O gives the energy cost of metal cans - not beverage cans; of motor vehicles - not Chevrolets.

The purpose of this paper is to present a procedure for combining the two methods to minimize the disadvantages of each. The combination is straightforward because both methods are linear and both require the same type of input data. Finally, this "hybrid" analysis technique, to systematically reduce uncertainty in the energy cost of a commodity, is applied for an example calculation.

The "Ideal" Process Analysis

Consider the most general case of an economic system in which there are N types of goods and services (N may be arbitrarily large,

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*In this paper the word technology is to be interpreted in a narrow sense, as describing material and energy inputs to production processes. A more detailed definition is given in the next section.

on the order of thousands or millions). In order to calculate the energy cost of one of these, a diagram or tree is constructed such as that in Fig. 1. There may be up to N inputs at each node and the number of nodes is, in principle, unlimited. At each node the direct energy input is tabulated; the energy cost of the good or service is the sum of these inputs.

For a typical product, n, the production technology is represented by a vector $\underline{a}_n$ where a typical element a_{in} represents the amount of product i needed directly to produce a unit of product n. The N x N matrix $\underline{\underline{a}}$ then provides a linear representation of the technology of producing all goods and services.

Let ε_n represent energy intensity of product n. It is given by:

$$\varepsilon_n = \delta_{en} + a_{en} + \sum_{i=1}^{N} a_{ei}\, a_{in} + \sum_{j=1}^{N} \sum_{i=1}^{N} a_{ej}\, a_{ji}\, a_{in} + \cdots \qquad (1)$$

where the subscript e denotes the energy sector and $\delta_{en} = 0$ for $n \neq e$; δ_{ee} represents the heat content of a unit of energy.* The term a_{en} denotes the energy used directly in producing a unit of product n, and succeeding terms correspond to direct energy inputs at each level of the process analysis tree.

*Energy is usually measured in terms of enthalpy, so $\delta_{ee} = 1$.

In practice, such a large number of terms is never computed. At the first level only the most significant inputs are considered, and, of those, only a subset is further broken down into its components. Unfortunately, diminishing contributions from each level provide no information for a truncation rule.*

Input-Output Analysis

This is a technique for representing the entire system of N production processes as a linear network model. Each node characterizes a sector of the economic system, each producing a unique good or service. Figure 2 shows the energy flows entering and leaving each sector.

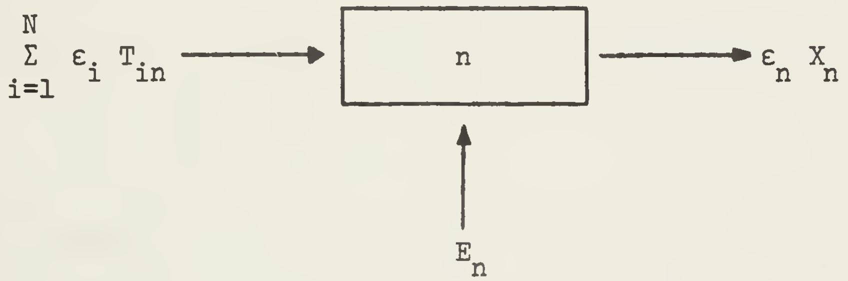

Figure 2. Energy Balance for a Producing Sector

Energy embodied in inputs from other sectors, $\varepsilon_i T_{in}$, enters at the left, while energy embodied in the sector's output X_n is shown

*Consider for example, the divergent series $\Sigma\frac{1}{n}$.

exiting at the right. If, in Fig. 2, sector n denotes the energy sector, an amount E_n is extracted from the earth. The energy balance equation becomes:

$$\sum_{i=1}^{N} \varepsilon_i T_{in} + E_n = \varepsilon_n X_n \tag{2}$$

or, in matrix notation we have:

$$\underline{\varepsilon}\, \underline{\underline{T}} + \underline{E} = \underline{\varepsilon}\, \hat{\underline{\underline{X}}} , \tag{3}$$

which is a set of N equations that can be solved for the N unknowns $\underline{\varepsilon}$. $\hat{\underline{\underline{X}}}$ is the diagonal matrix whose elements represent the total output from each sector.

From the definition of $\underline{\underline{a}}$ given earlier, we have:

$$\underline{\underline{T}} = \underline{\underline{a}}\, \hat{\underline{\underline{X}}} \tag{4}$$

and eq. (3) becomes:

$$\underline{\varepsilon} = \underline{e}\, (\underline{\underline{I}}-\underline{\underline{a}})^{-1} \tag{5}$$

where $\underline{e}$ is a unit vector which identifies the energy sector row of $(\underline{\underline{I}}-\underline{\underline{a}})^{-1}$ as the energy intensities.

Combining Process and Input-Output Analyses

It is now clear that the energy cost of any good or service can be determined by either process analysis (eq. 1) or input-output analysis (eq. 5). Both methods are linear and require identical input data. The proof that the results are identical is straightforward.

If the spectral radius of $\underline{\underline{a}}$ is less than unity,* we have

$$\underline{\varepsilon} = \underline{e} \; [\underline{\underline{I}} + \underline{\underline{a}} + \underline{\underline{a}}^2 + \underline{\underline{a}}^3 + \ldots.] \tag{6}$$

which shows that the energy intensities calculated using eq. (5) are identical to those obtained through the more tedious process analysis eq. (1).

For most applications, however, this rather complete set of data (the N x N matrix $\underline{\underline{a}}$) is not available at the necessary level of detail. It exists only at a more aggregated level K<N, where K≃360 for the United States economic system, and is much smaller for most other nations.

Because of this lack of data, input-output results give only the average energy intensity of a sector's output. Accuracy is limited by the level of aggregation: the energy intensity of "metal cans" would apply to both aluminum beer cans and 55-gallon oil drums.

Process analysis does provide a framework for obtaining new detailed problem-specific data. For example, the energy consumed

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*This condition is indeed satisfied for the 360-sector description of the U.S. economic system.

directly by the beer can manufacturer is the second term in eq. (1). However, the process analysis must soon be terminated as the data acquisition effort rapidly increases and certain data are found to be unavailable.

Fortunately, the truncation error can be minimized using the results of input-output analysis. Note that the series expansion in eq. (1) can be truncated at any level.

$$\underline{\varepsilon} = \underline{e}\,[\underline{\underline{I}} + \underline{\underline{a}} + \underline{\underline{a}}^2 + \cdots + \underline{\underline{a}}^m\,(\underline{\underline{I}}-\underline{\underline{a}})^{-1}]\ . \tag{7}$$

It is trivial to prove the matrix product in the last term commutative, so the energy intensities are given by:

$$\underline{\varepsilon} = \underline{e}\,[\underline{\underline{I}} + \underline{\underline{a}} + \underline{\underline{a}}^2 + \cdots + \underline{\underline{a}}^{m-1}] + \underline{\varepsilon}'\underline{\underline{a}}^m \tag{8}$$

where $\underline{\varepsilon}'$ is a vector of energy intensities described in more detail below. To compute the energy cost of a particular item, one may evaluate the first few terms from available (problem-specific) data and, in doing so, truncate the expansion early. Ultimately, the entire matrix $\underline{\underline{a}}$ would be filled as the number of steps m increases. It is filled columnwise, as the technology for producing each input is specified. Typically, the process is terminated at m=2 or 3, before too many inputs are involved.

The last term in eq. (8) approximates the sum of the truncated terms. The vector $\underline{\varepsilon}'$ is obtained from an input-output analysis of the economic system described at the K-sector level of detail, where K<N.

$\underline{\varepsilon}'$ is an N-order vector containing only K distinct values. The order N, required for eq. (8), is obtained by repeating each of the K distinct energy intensities a number of times. The error involved in this truncation depends on the uncertainty introduced by characterizing these m^{th} level inputs as "typical" outputs of industries aggregated to the K-sector level.

In summary, process analysis provides a framework for utilizing a limited amount of problem-specific data to reduce the "aggregation error" inherent in input-output results. Using I-O results to truncate the analysis eliminates the problem of an unknown truncation error, replacing it by a smaller aggregation error associated with energy-costing the higher order inputs. We shall call the combination of these techniques "hybrid analysis" and describe the procedures below.

Error Reduction Criterion

In practice, each step in a process analysis may be viewed as an expansion of the system boundary (around the item being analyzed) into the economic system, tabulating direct energy inputs at each step (see Fig. 3). The results of input-output analysis may be used to estimate the energy embodied in flows crossing the system boundary at any level, by associating each good or service with one of the $K<N$ sectors of the I-O model.

The I-O results are indifferent to the location of the system boundary interface. Regardless of the number of process analysis steps taken, the system boundary still looks the same from the I-O side; only

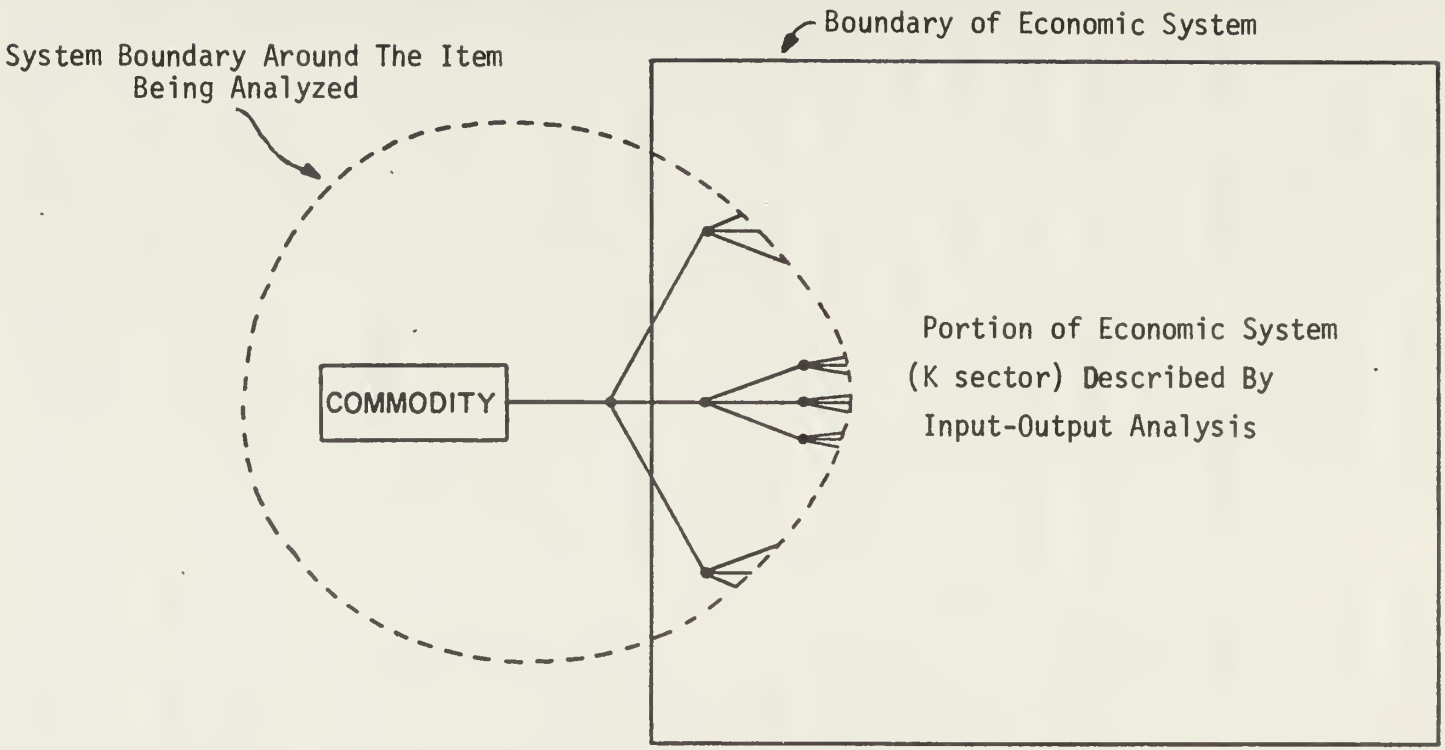

Figure 3. System Boundaries for Process and Input-Output Analyses

K types of goods and services cross the boundary. The energy cost of each of those is known.

For industrial sector i, ε_i' is the energy intensity (eq. 8) and α_i is the amount of product i crossing the boundary. The energy embodied in the flow of sector i goods is therefore $\varepsilon_i'\alpha_i$. This flow term is analogous to the truncation term in eq. (8) for energy intensities. In general, ε_i' and α_i have uncertainties $\Delta\varepsilon_i'$ and $\Delta\alpha_i$, respectively.*

The criterion for optimizing the hybrid analysis procedure is to minimize uncertainty in total energy costs. For this purpose, inputs from the K-sector economic system at each process analysis step are classified into one of three categories: Type 1; typical of a sector's output, Type 2; atypical of a sector's output, or Type 3; miscellaneous (not specified as output from a particular sector). For atypical inputs, the analyst must increase energy intensity uncertainties to $\Delta\hat{\varepsilon}_i' > \Delta\varepsilon_i'$, reflecting the aggregation error. Type 3 inputs are assigned an average energy intensity designated as ε_o' with uncertainty $\Delta\varepsilon_o'$.**

Obviously, the total uncertainty $(\varepsilon'\Delta\alpha + \Delta\varepsilon'\alpha + \Delta\varepsilon'\Delta\alpha)$ can be reduced by reducing the $\Delta\alpha$ term. Uncertainties in energy intensities (ε') of Type 2 or 3 inputs can usually be reduced by a process analysis of their inputs.*** Figure 4 shows the first-order errors associated with a Type 2 or 3 input disaggregation. A step-by-step procedure to

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*For a discussion of the $\Delta\varepsilon_i'$ uncertainties see Bullard and Sebald (1975).

**Usually, the value used here is that of the energy/GNP ratio if inputs are measured in dollars.

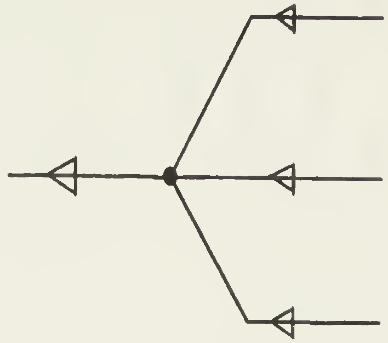
***Because $\Delta\alpha$'s may increase as disaggregation continues, minimum uncertainty will not necessarily be achieved by greater disaggregation.

<u>INITIAL</u>

Type 2: $\varepsilon'_i\alpha_i + \varepsilon'_i\Delta\alpha_i + \Delta\hat{\varepsilon}'_i\alpha_i$

or

Type 3: $\underbrace{\varepsilon'_o\alpha_o}_{\text{Starting Value}} + \underbrace{\varepsilon'_o\Delta\alpha_o + \Delta\varepsilon'_o\alpha_o}_{\text{First Order Error}}$



<u>DISAGGREGATED</u>

$$\sum_{\text{Type 1}} \varepsilon'_j\alpha_j + \sum_{\text{Type 1}} (\varepsilon'_j\Delta\alpha_j + \Delta\varepsilon'_j\alpha_j)$$

$$\sum_{\text{Type 2}} \varepsilon_j\alpha_j + \sum_{\text{Type 2}} (\varepsilon'_j\Delta\alpha_j + \Delta\hat{\varepsilon}_j\alpha_j)$$

$$\underbrace{\varepsilon'_o\alpha_o}_{\text{Improved Value}} + \underbrace{\varepsilon'_o\Delta\alpha_o + \Delta\varepsilon'_o\alpha_o}_{\text{First Order Error}}$$

Figure 4. Energy Flows for Type 2 or Type 3 Disaggregation.

reduce the overall uncertainty is demonstrated by a simplified example in the Appendix.

SUMMARY

To reduce the uncertainty that is unavoidable in either of the conventional energy analysis methodologies, the methods can be combined to minimize the drawbacks of each. Process analysis alone is intractable because of the massive data requirements needed for completion; a truncated process analysis has an unknown error. Input-Output analysis, on the other hand, is blessed with a global data base but is severely constrained by aggregation problems and an outdated description of technology.

Using the combined "hybrid" analysis technique presented here, errors can be minimized and quantified. Depending on the data obtainable, the complexity of the analysis can be traded off against the accuracy desired.

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APPENDIX: A THREE SECTOR EXAMPLE

This section illustrates the hybrid analysis procedure for energy costing "widgets" using a hypothetical three-sector economy. The table below gives the necessary I-O data for the analysis. Figure 5 shows the dollar flows and system boundary definitions for each level of the analysis. (For simplicity, only positive errors are considered.)

INPUT-OUTPUT DATA FOR A HYPOTHETICAL THREE SECTOR ECONOMY

Sector	Energy Intensity (Btu/$)	Uncertainty[a] (Btu/$)
1	4.0	.5
2	7.0	1.0
3	15.0	1.0
Miscellaneous[b]	8.0	10.0

[a]Only positive error terms are considered here.

[b]Corresponds to Type 3 inputs.

Step I. The initial step evaluates the energy costs and uncertainties at the most aggregated level of analysis. At this level inputs are all considered to be of Type 3. The total dollar costs and

uncertainties are found (e.g., from a widget expert) to be as follows:

$$\alpha_o = 10.0 \qquad \Delta\alpha_o = 1.0$$

The resulting energy costs and associated first-order (positive) error becomes:

$$\text{Energy Cost} = \Sigma\ \varepsilon\ \alpha = (8.0)(10.0) = 80.0 \text{ Btu}$$

$$\begin{aligned}\text{Positive Error Term} = \ & \Sigma(\varepsilon\Delta\alpha + \Delta\varepsilon\alpha) = (8.0)(1.0) + (10.0)(10.0) \\ & = 8.0 + 100.0 = 108.0 \text{ Btu}\end{aligned}$$

At this level of analysis the error term is quite large and dominated by the component $\Delta\varepsilon_o'$. This error can be reduced by identifying individual inputs to the widget.

<u>Step II(a)</u>. The analyst obtains additional information to classify inputs to the widget by sector. Requirements from sectors 1 and 3 are found to be typical but those from sector 2 are felt to be atypical. There are still some requirements that remain unclassified. The sector input values and uncertainties are found to be:

$$\begin{array}{lll} \alpha_1 = 2.0 & \Delta\alpha_1 = .1 & \\ \alpha_2 = 3.9 & \Delta\alpha_2 = .1 & \Delta\hat{\varepsilon}_2' = 2.0 \text{ (Note: } \Delta\varepsilon_2' = 1.0) \\ \alpha_3 = 4.0 & \Delta\alpha_3 = .8 & \\ \alpha_o = 0.1 & \Delta\alpha_o = .1 & \end{array}$$

New estimates of energy costs and first order uncertainties are obtained (inputs ordered in counterclockwise progression from Fig. 4).

$$\text{Energy Costs} = \Sigma\varepsilon\alpha = (4.0)(2.0) + (15.0)(4.0) + (7.0)(3.9) + (8.0)(.1) = 8.0 + 60.0 + 27.3 + .8 = 96.1 \text{ Btu}$$

$$\begin{aligned}\text{Positive Error Term} &= \Sigma(\varepsilon\Delta\alpha + \Delta\varepsilon\alpha) \\ &= [(4.0)(.1) + (.5)(2.0)] + [(15.0)(.8) + (1.0)(4.0)] + [(7.0)(.1) + (2.0)(3.9)] + [(8.0)(.1) + (10.0)(.1)] \\ &= \underset{(\text{sector } 1)}{(.4 + 1.0)} + \underset{(\text{sector } 3)}{(12.0 + 4.0)} + \underset{(\text{sector } 2)}{(.7 + 7.8)} + \underset{(\text{misc.})}{(.8 + 1.0)} \\ &= 27.7 \text{ Btu}\end{aligned}$$

The error term is nearly a fourth of the one obtained in the first step, but is still about 25% of the calculated energy cost. The major component of error is due to the $\Delta\alpha$ term for sector 3 inputs. This error can be reduced by returning to the data source for a better estimate of sector 3 inputs.

<u>Step II(b)</u>. The analyst obtains an improved sector 3 input specification. Step II(a) is repeated except that $\Delta\alpha_3$ has been reduced from .8 to .1 (the α_3 estimate is found to be the same as before). The resulting energy cost and uncertainty are calculated:

Energy Costs = 96.1 Btu (same as Step II(a))

Positive Error Term = $27.7 - \underset{\text{old}}{12.0} + \underset{\text{new}}{1.5} = 17.2$ Btu

Now that the 12.0 error term is reduced to 1.5 the major contributor to the uncertainty is due to $\Delta\hat{\varepsilon}_2'$ (the 7.8 term). This error component can be reduced by specifying the inputs required to produce the atypical sector 2 output.

Step III. The analyst disaggregates the inputs into the atypical sector 2 output. The following typical inputs are found:

$$\alpha_1 = .1 \qquad \Delta\alpha_1 = .01$$
$$\alpha_2 = 3.8 \qquad \Delta\alpha_2 = .1$$

Note from Fig. 5 that there are now five inputs crossing the system boundary. Using the counterclockwise accounting scheme, the first two and last one are identical to the previous step. Incorporating the new information,

$$\begin{aligned} \text{Energy Costs} = \Sigma\varepsilon\alpha &= (4.0)(2.0) + (15.0)(4.0) + \\ &(4.0)(.1) + (7.0)(3.8) + (8.0)(.1) = 8.0 + \\ &60.0 + .4 + 26.6 + .8 \\ &= 95.8 \text{ Btu} \end{aligned}$$

$$\text{Positive Error Term} = \Sigma(\varepsilon\Delta\alpha + \Delta\varepsilon\alpha)$$

$$= [(4.0)(.1) + (.5)(2.0)] + [(15.0)(.1) + (1.0)(4.0)] + [(4.0)(.01) + (.5)(.1)] + [(7.0)(.1) + (1.0)(3.8)] + [(8.0)(.1) + (10.0)(.1)]$$

$$= \underset{\text{(sector 1)}}{(0.4 + 1.0)} + \underset{\text{(sector 3)}}{(1.5 + 4.0)} + \underset{\text{(sector 1)}}{(.04 + .05)} + \underset{\text{(sector 2)}}{(.7 + 3.8)} + \underset{\text{(misc.)}}{(.8 + 1.0)}$$

$$= 13.29 \text{ Btu}$$

The uncertainty is now reduced to an eighth of that in the initial step. The major components of error are due to $\Delta\varepsilon'$ error terms for the sector 3 inputs (the 4.0 term) and the (typical) sector 2 input into the atypical sector 2 requirement (the 3.8 term). Further reduction in these uncertainties can only result from an improved I-O specification. Therefore, this seems an appropriate point to terminate the analysis. The resulting energy cost of the widget becomes:

$$\text{Energy Cost} = 95.8 + 13.29 \text{ Btu.}$$

(In an actual analysis negative uncertainties would be treated in a manner identical to the positive errors considered in this example.)

The table below summarizes the sample case described above. Although this example is only hypothetical, note that the reduction in uncertainty decreases rapidly as the analysis progresses.

TABLE 1. Summary of three-sector example.

STEP	ACTION TAKEN	ENERGY COST (Btu)	UNCERTAINTY (Btu)	UNCERTAINTY REDUCTION (Btu)
I	-	80.0	108.	-
II(A)	Process analysis of widget inputs	96.1	27.7	81.3
II(B)	Tighten $\Delta\alpha_3$	96.1	17.2	10.5
III	Process analysis of atypical sector 2 inputs	95.8	13.3	3.9